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ON CONFIGURATIONS INVOLVING THREE SHOCKS THROUGH ONE POINT MODIFIED BY MEYER WAVES

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A MEMORANDUM SUBMITTED
by the
APPLIED MATHEMATICS GROUP, NEW YORK UNIVERSITY
to the
APPLIED MATHEMATICS PANEL
NATIONAL DEFENSE RESEARCH COMMITTEE

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September, 1943

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ON CONFIGURATIONS INVOLVING THREE SHOCKS THROUGH ONE POINT MODIFIED BY MEYER WAVES.

By K. O. Friedrichs

Following upon a discussion with John von Neumann, the writer made an investigation of possible configurations involving three shocks through one point and Mayer waves. A variety of such configurations was found but so far it does not seem that these modified Mach configurations can serve to explain a continuous transition of regular reflection into Mach reflection for weak shocks.

I. Remarks on Various Configurations.

1. Consider three shock configurations (Fig. 1) in which the air crossing the reflected shock turns toward the discontinuity line and the region past the reflected shock is supersonic. (The manifold of such configurations is adjacent to those with glancing incidence; it corresponds to the cross-hatched region in Fig.2). In the neighborhood of these three shock configurations there is a variety which has, in addition, a Meyer wave in region (2). This Meyer wave may be a rarefaction wave, a compression wave, or begin with compression and end up as rarefaction. In any case, the air flows through this wave from the region (2') past the reflected shock to the region (2") adjacent to the discontinuity line.

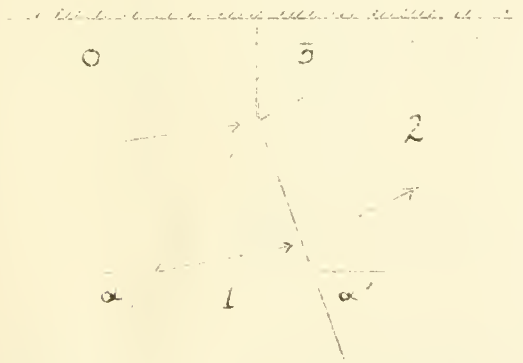

Fig. 1

Clearly, none of these configurations is "stationary".

The angles α in these configurations will all be near to 90°, it seems. On the other hand, it seems, one may prescribe the state (0), further p_1, p_2 and the angle between incident and reflected shocks (θ is probably

Fig 2

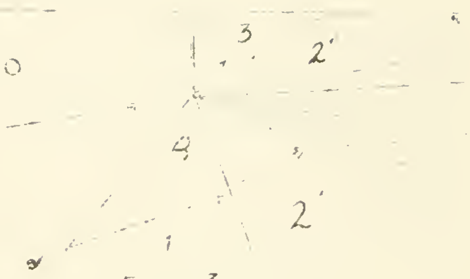

Fig 3

restricted), then there will be two solutions, one in which the Meyer wave begins as rarefaction, the other in which it begins as compression. If one lets β approach zero one obtains the case of one shock overtaking another, in which case, as v. N. had shown, a rarefaction wave is reflected.*

The described configurations will also occur in the following problem. Consider a circular shock wave which overtakes a plane shock. After interaction a configuration as indicated in Fig. 4 will probably result.

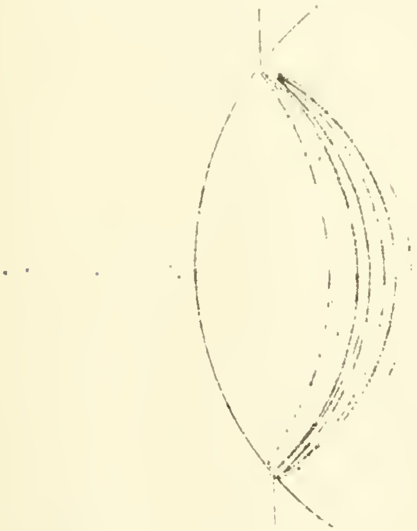

Fig. 4

When the flow is observed from one of its branch points it will in its neighborhood coincide with one of the flows just described.

2. Von Neumann conjectured that stationary Mach and regular reflection flows might be connected through flows in which additional waves occur between incident and reflected shock. (Cf. Fig. 5.) The writer found a variety of flows having a Meyer wave in the sector between incident and reflected shock. This Meyer wave always begins as a compression wave; it may end up as a rarefaction. These flows are possible in the neighborhood of any three-shock flow. Suppose the incident shock and the Meyer wave, i.e., the states before and past, are prescribed. Then there will be either two solutions or none (or, of course, one solution counting double). If one lets

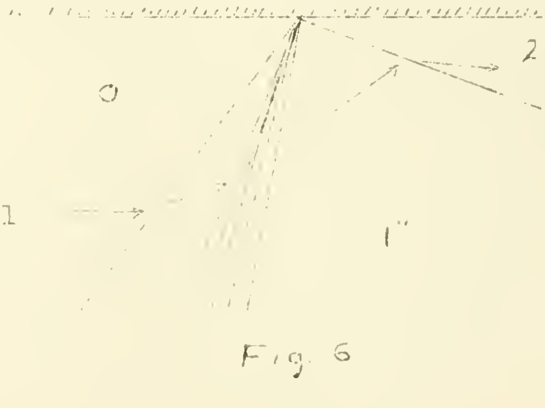

Fig. 6

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* The configuration in which the Meyer wave begins as compression degenerates in the limit $\beta \to 0$.

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the Meyer wave shrink to zero, one of these two solutions approaches the three-shock solution, the other approaches one in which the reflected shock is sonic. (By the way, near $\alpha = 90^\circ$ there are flows for which the reflected shock has disappeared.)

Among the flows so described there is also a manifold of stationary configurations. It is to be noted, however, that they all occur with angles α less than those for an unmodified stationary Mach configuration.

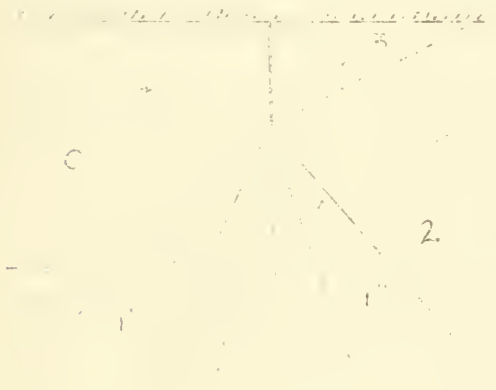

Fig. 6

Cf. Fig. 6. It is interesting to note that regular reflection can also be modified by including a Meyer wave between incident and reflected shock. This may be done without changing the incident shock, i.e., p_1/p_0 and α; (only the strength of the Meyer wave is restricted); the reflected pressure then increases somewhat. On the other hand, one can show that these modified regular reflections may exist only when simple regular reflection is also possible for the same incident shock. It is true that among these modified regular reflections there are some which are stationary Mach flows if the incident pressure ratio p_1/p_0 is somewhat less than 2.31, the reflection may even be weak while the unmodified stationary Mach reflection would be strong. Thus they might furnish a bridge between regular and Mach reflection for p_1/p_0 somewhat below 2.31. However, the experimental results which von Neumann has shown us are definitely against this possibility.

4. Another possibility comes to mind, viz. that the Mach shock has a compressive Meyer wave as a forerunner (see Fig.7). (This would confirm another of von Neumann's conjectures, viz. that something happens in front of the Mach shock.) It can be shown that to every regular reflection configuration given by (0), (1), (2) (cf. Fig. 7), there is one of these Mach configurations with a forerunner having the same states (0), (1), (2). There are some counter-arguments, however.

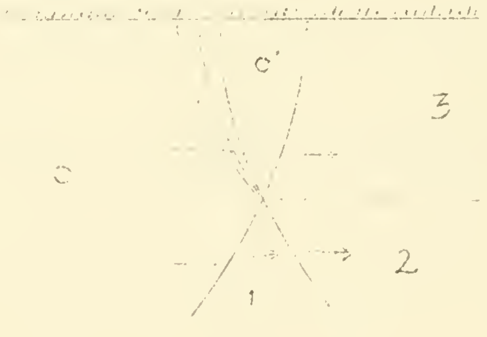

Fig. 7

Firstly, the angle between the Mach shock at the branch point and the wall would deviate considerably from 90° in contradiction to experimental evidence. This difficulty might perhaps be overcome somehow or other. At least the Mach line shows in the direction in which it is mostly observed. Another difficulty seems to be more serious. A compressive Meyer wave should not turn as indicated in Fig. 7 and enter the wall perpendicularly, for a compressive wave should contract when it is crossed perpendicularly. Thus, even the proposition of a compressive forerunner is not too hopeful.

II. Justification of Above Remarks.

We shall use the (p,θ)-diagram (as in the Appendix to the writers remarks on the Mach effect), θ being the angle of inclination of a velocity vector, observed from the branchpoint.

All states (p, θ) that can be connected with (p_0, θ_0) through an oblique shock then lie on a "shock" curve (Fig.8).

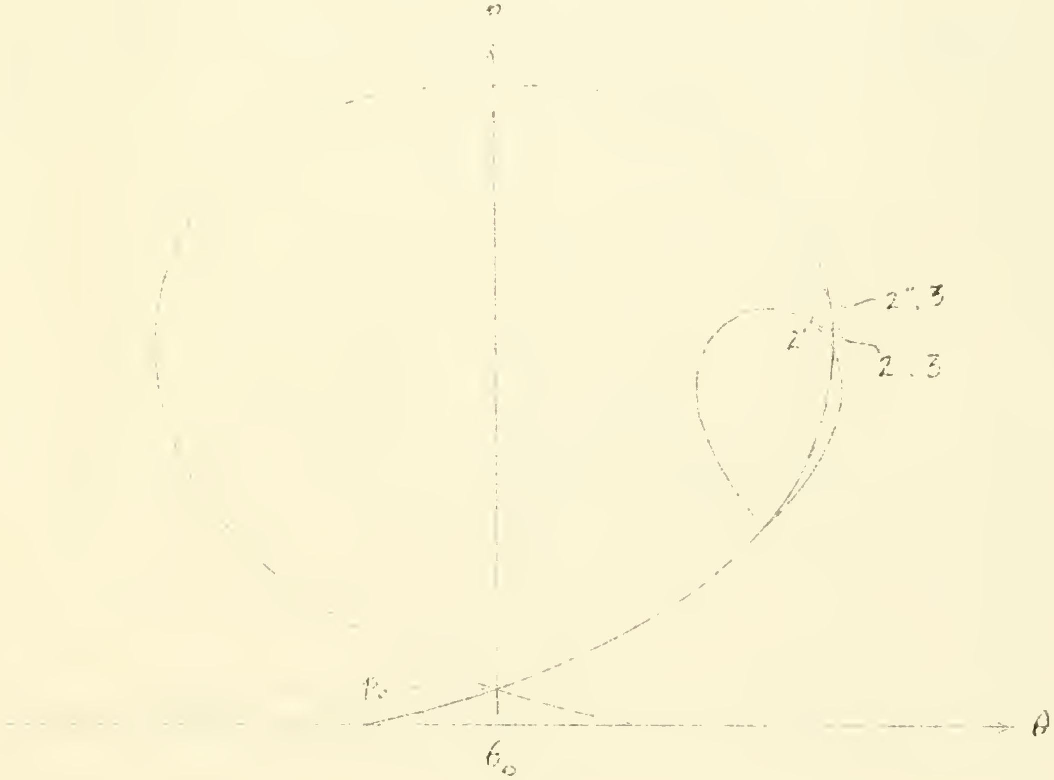

Figure 8.

1. Let (1) be a point on this curve and draw the shock curve through (1). Its intersection with the 0-curve gives a three shock configuration. If the angle of incidence α is nearly 90°, the situation is as in Fig.8. ($\alpha = 90^\circ$ would be reached when both curves touch each other in (1)). If α is sufficiently near to 90° the state given by the intersection on the (1)-curve will be supersonic; the same will then be true for points on the (1)-curve above the intersection. Take any of these points (2'). Through (2') draw the curve corresponding to all states that can be connected with (2') through a Meyer wave with increasing angle θ. Such a curve is the image of a characteristic. (There are two such characteristics, one beginning as compression, the other as rarefaction.) The intersection of any of these two curves with the (0)-curve yields a state (2") or (3) respectively, which leads to the configuration as described in Fig.3.

2. To insert a Meyer wave between incident and reflected shock one considers a (0)-curve, on it a point (1') corresponding to supersonic flow. Through (1') one places the compressive branch of the Meyer curve and on it any point (1") with $\theta_{1''} > \theta_{1'}$. The shock-curve through (1") then intersects the (0)-curve in a point corresponding to states (2) and (3) respectively, cf.Fig. 5.

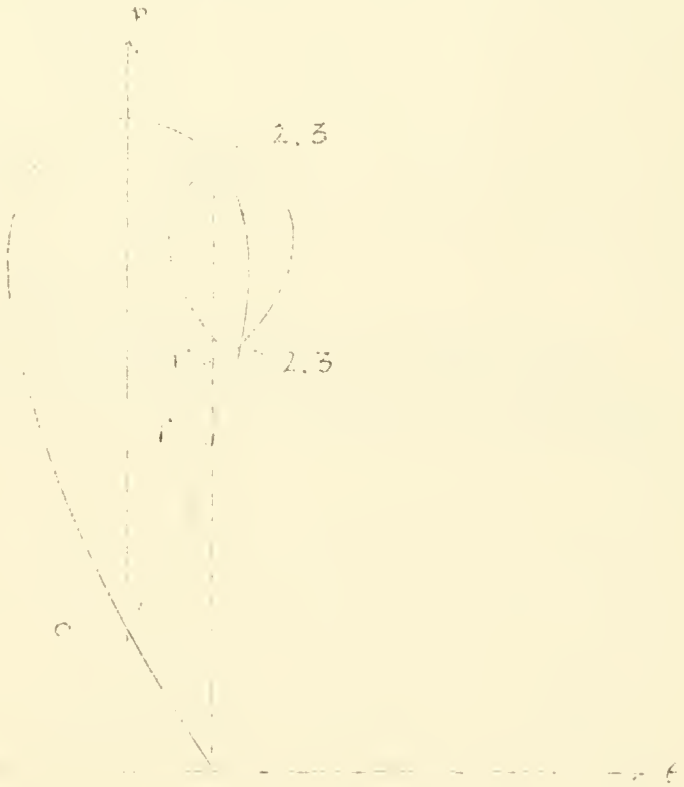

Figure 2.

To obtain a stationary configuration one must begin with a state (1') that would lead to an inverted three-shock configuration and thus have an angle α less than the stationary three-shock configuration for the same p_1/p_0. Cf. Fig.10.

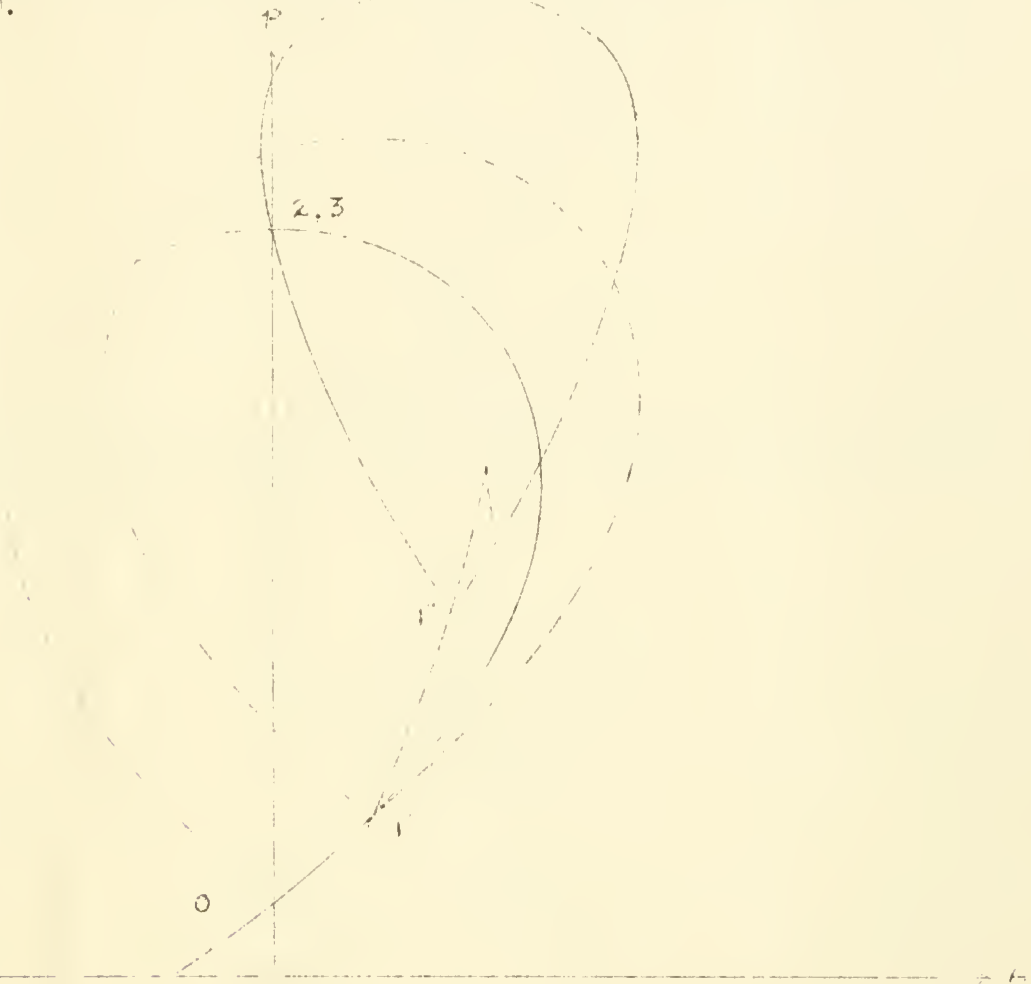

Figure 10.

5. The possibility of a regular reflection modified by a Meyer wave between incident and reflected shock can be seen from similar arguments, cf.Fig.11.

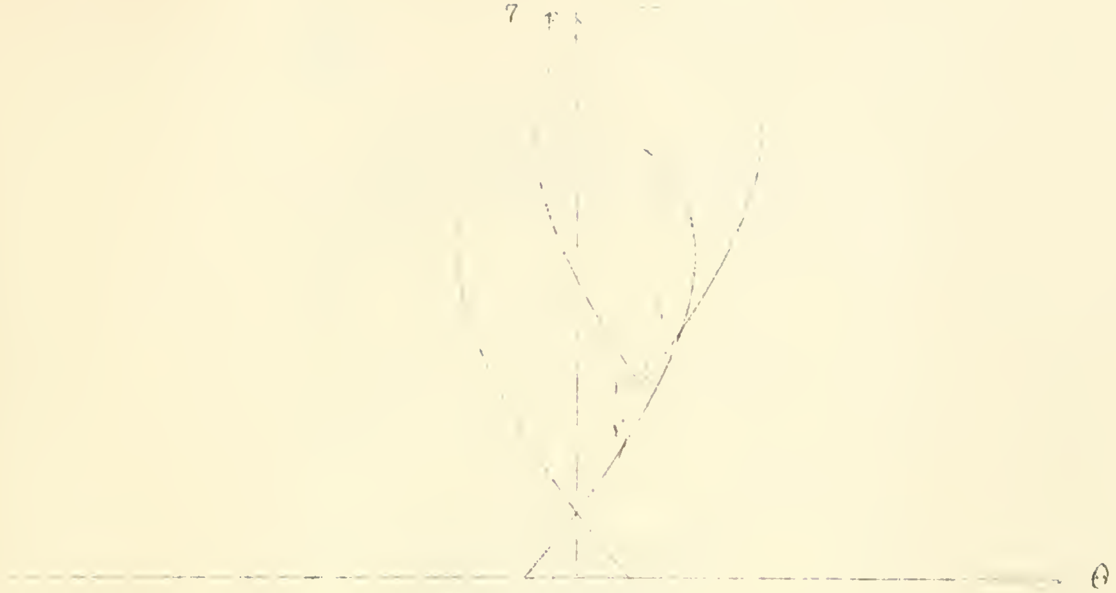

Figure 11.

. The local possibility of a compressive forerunner of the Mach shock is also easily seen, cf.Fig.12.

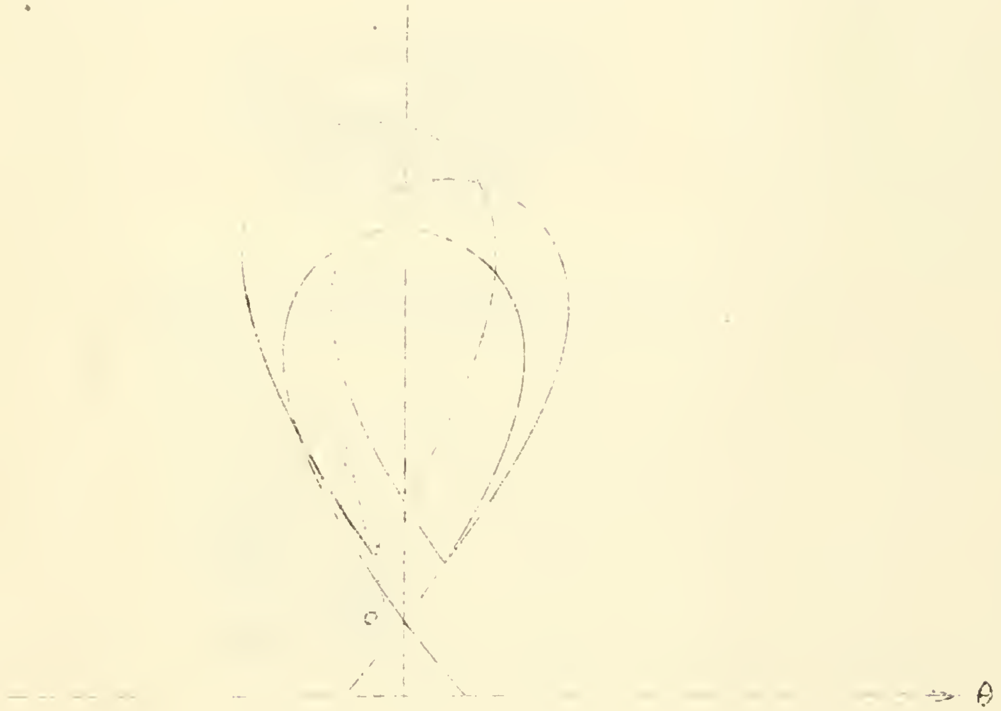

Figure 12

Clearly, a shock forerunner would also be possible. It would even seem more natural.

9. Here one might make a general remark. If there is continuous transition from regular to Mach reflection, it seems likely that new phenomena occur only between branch point and wall. If Meyer or other waves occur in the regions (1) or (2), e.g., they should already have been there for regular reflection.

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